Texas Mathematics Teacher

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A PUBLICATION OF THE TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

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INVESTIGATING MONOPOLY
STATE UPDATE
AWARDS
TEXAS COUNCIL OF TEACHERS OF MATHEMATICS
GOALS 2002-2003

MISSION: To promote mathematics education in Texas.

GOALS:

• Collect e-mail addresses from all members for electronic communication
• Maintain journal and website quality
• Support the Conference for the Advancement of Mathematics Teaching (CAMT) with volunteers, sponsor and staff TCTM booth at CAMT
• Award college scholarships and recognize service, increase scholarship funding
• Improve communication with affiliated groups in Texas, advertise their conferences on the TCTM website and in the journal
• Increase membership via CAMT and encourage affiliated groups to put TCTM membership on their membership forms
• Increase public relations efforts

TCTM Past-Presidents

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*Texas Mathematics Teacher*, the official journal of the Texas Council of Teachers of Mathematics, is published in the fall and spring. Editorial correspondence and manuscripts should be mailed or e-mailed to the editor.

All readers are encouraged to contribute articles and opinions for any section of the journal.

Manuscripts, including tables and figures, should be typed in Microsoft Word or Works. Submit four copies and an IBM formatted 3 1/2 inch diskette containing the manuscript or send as an e-mail attachment to the editor.

Articles for *Voices From the Classroom* should be relatively short. A discussion of appropriate grade level and prerequisites for the lesson should be included.

Items for *Lone Star News* include, but are not limited to, TCTM affiliated group announcements, advertisements of upcoming professional meetings, and member updates.

Businesses interested in placing an advertisement for mathematics materials should contact Mary Alice Hatchett.
TEA Talks
Paula Gustafson, Director or Mathematics, pgustafs@tea.state.tx.us

This column provides TCTM members with the latest information from the Texas Education Agency about the Texas Essential Knowledge and Skills (TEKS), educational policy, and, of course, the TAKS test. Refer to the websites listed for additional information on each topic.

Hot News

••Algebra EOC is available for district use on a voluntary basis beginning in December 2002. An online assessment will be available through NCS, with a hard copy version also available for districts to administer and score. www.tea.state.tx.us/student.assessment/resources/letters/algebra1.pdf

••The TMDS (Texas Mathematics Diagnostic System) is now available for use with students in grades 5-8. This diagnostic assessment is provided by legislative appropriation in HB 1144. The diagnostic will measure students’ algebra readiness and can be administered online or in hard copy. During later phases of development, teachers will be able to develop their own assessments by accessing an item bank aligned to all TEKS student expectation statements in grades five through eight. Districts will need to register and send data to Vantage Learning prior to accessing the system. www.accesstmds.com

••The State Board of Education has adopted the TAKS passing standards for 2003-2005. The good news for educators is the adoption of a phase-in model that provides campuses the time necessary to gear-up curriculum and student understanding in mathematics before reaching the panels’ recommendations at each grade level. www.tea.state.tx.us/press/taksapprov.html

••Region IV in Houston has created performance assessments for students in grades three through Algebra II. This project was funded by TEA and includes assessment items that assist teachers in determining how well students are performing within given concepts. The multiple representations and solution strategies that students must use while solving these tasks will prepare them for the rigorous content and expectations on TAKS. www.mathbenchmarks.org

••TAKS blueprints outlining the number of items per objective are available for educator use. These blueprints provide educators with an understanding of the actual weighting of the test by objective, and include the total number of items that will be scored. The TAKS test this spring will include field test items at all grade levels. www.tea.state.tx.us/student.assessment/taks/blueprints/index.html
• Texas Instruments has now placed the Test Guard Program in VIP accounts. This move will allow educators to download the program quickly without some of the difficulties previously encountered. This program will allow teachers in grades 9-11 to quickly delete all applications and data stored on graphing calculators prior to TAKS testing.


[Editor’s Note: If this link does not open for you, please use the following path: Texas Instruments homepage, Educators, Math & Science, local support (more), Texas, TI TestGuard™ App for the TI83 Plus.]

• Nomination forms for the Presidential Awards for Excellence in Mathematics and Science Teaching are now available. The award for this year will be available for math and science teachers in grades 7-12. Nomination forms must be submitted to me at the Agency prior to the application being sent to qualified candidates. E-mail me at pgustafs@tea.state.tx.us if you would like to nominate a colleague.

Keep in Mind

• The Agency will not release another version of the Information Booklet prior to the Spring 2003 TAKS test.

• A TAKS study guide will be provided for students that fail any portion of the test. The format will be similar to previous TAAS study guides.

• The Spring 2003 TAKS test will be released on the TEA website in mid-August.

• The SDAA (State Developed Alternative Assessment) will still be available. The format of this test is currently under revision to align objectives with the TAKS. Revisions will not be complete until the 2004-2005 school year.

• Many activities, lessons and assessments are available free of charge on the Mathematics Toolkit. If you are searching for TEKS-based materials, this is the website you want to visit! www.mathtekstoolkit.org

• Research shows long-term, intensive staff development is most effective in changing current practice. Texas is fortunate to have the TEXTEAMS staff development program available for all educators. Check with your local ESC mathematics contact to determine when the next institute will be held. Information about the TEXTEAMS Institutes can be found at www.texteams.org.

• Teachers at all grade levels should utilize technology for solving rich, challenging mathematics problems. This becomes increasingly important in the high school years as students will be using graphing technology on all high school TAKS mathematics assessments.

• A list serve is provided for educators interested in receiving e-mail updates about mathematics events. To join the list visit the following website and follow the prompts.

www.tea.state.tx.us/list/
Dear TCTM Members,

Let me begin by apologizing for the delay in publishing and mailing the Fall journal. Due to personnel changes on the journal staff and illness of the current staff, it was delayed excessively. I promise to have the Texas Mathematics Teacher to you on time in the future. We are seeking a journal assistant to work on the publication. If you are interested, please contact me at 512-475-9713 or cschneider@mail.utexas.edu.

Mathematics education in Texas is continuing to change to meet the demands of our new accountability assessment. My goals as president are to improve communication electronically with all members. Please send me a message so I can add you to the database. My e-mail is posted above. In this fashion I hope to bring our membership more timely messages about state activities and opportunities relevant to your work.

The work of our organization continues, in spite of the delay of this journal. We will be collecting scholarship, leadership award, and CAMTership applications in May. We will continue to support the Conference for the Advancement of Mathematics Teaching (CAMT) this summer with dozens of volunteers. With your help we will support and communicate the activities of our local affiliate groups. These are the main responsibilities of our organization.

I would like to encourage all members to become more active in their local affiliates and to speak out on issues concerning mathematics education. President Bush will be announcing a national mathematics initiative in the future and it is important that effective, experienced teachers provide the feedback necessary to make such an initiative successful. We here in Texas continue to model many of the pieces of President Bush’s education plan. Our feedback as participants is critical to improve implementation.

One place to make your voice heard is at our national conference which this year is in San Antonio. Please plan to attend. You may find information at www.nctm.org under annual conference. TCTM is a co-sponsor this year and we have been working behind the scenes to make this a memorable conference. We are sure that the facilities will provide better access to sessions.

Finally, I recommend reading Helping Children Learn Mathematics published by the National Research Council in 2002. This is a synthesis of research on mathematics learning and teaching. It includes a description of what it means to be successful in mathematics. By interweaving five strands of mathematical proficiency, the authors have presented a balanced picture of what students should know and be able to do that will satisfy many differing views on this topic. The proficiencies, quoted verbatim from page 9, include:

• Understanding: Comprehending mathematical concepts, operations and relations—knowing what mathematical symbols, diagrams, and procedures mean.

• Computing: Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.

• Applying: Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.

• Reasoning: Using logic to explain and justify a solution to a problem or to extend from something known to something not yet known.

• Engaging: Seeing mathematics as sensible, useful, and doable—if you work at it—and being willing to do the work.

The remaining parts of this booklet outline how school mathematics needs to change, and what each of the various stakeholders (teachers, parents, administrators and policy makers) can do to support the necessary changes in school mathematics. You may read this document online at http://www.nap.edu/catalog/10434.html. A more comprehensive reference is Adding It Up: Helping Children Learn Mathematics which is also found online at http://www.nap.edu/catalog/9822.html.

The remaining months of this school will be a challenge to all Texas teachers. The stress of a new statewide test may be reduced by focusing on the needs of our students and support for our colleagues. I look forward to hearing about your successes at the end of the year.

Sincerely,

Cynthia L. Schneider
TCTM President 2002-2004
If your label includes a date earlier than “7/02,” please send the form on the next page to renew your TCTM membership. Don’t miss a copy of the Texas Mathematics Teacher.

Book Review
Navigating through Geometry in Prekindergarten-Grade 2

This publication is designed to assist the classroom teacher in effectively introducing geometry skills and concepts to students in prekindergarten–grade 2. Speech-language specialists might also find this publication useful because it emphasizes positional and directional words. The book contains nearly twenty activities covering two- and three-dimensional shapes, location and position, transformations and symmetry, visualization, spatial reasoning, and modeling. Blackline masters are included for making needed materials. The accompanying CD-ROM features five applets that allow students to explore topics using technology, along with articles from NCTM publications to develop the teacher’s background knowledge. Furthermore, Navigating through Geometry directly addresses the NCTM standards.

Many of the activities call for students to experiment with manipulatives, such as pattern blocks, attribute blocks, and mirrors. For children who are functioning at the most concrete level of thought, such hands-on experiences will enhance learning. Language and literature are appropriately incorporated into the activities for today’s whole-language classrooms. Teachers might consider including selected activities as free-time or learning-center options. Challenging concepts are introduced, but these ideas will ultimately form a solid foundation for geometry skills. Students may need more than one exposure to these lessons, and additional support may be necessary for some children. This high-quality publication is an excellent choice for teachers who are looking to incorporate fun and challenging geometry activities in the classroom.—Susan M. Eith, Gloucester Township Schools, Blackwood, NJ 08012.


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TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

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TCTM awards scholarships to high school seniors planning to pursue a career in mathematics teaching either as a mathematics specialist in elementary school or as a secondary school teacher with certification in mathematics. Your contributions in any amount are greatly appreciated. Please write a separate check for scholarship donations.

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Tired of Using Worksheets?

Dr. Mary Margaret Capraro, Assistant Professor, Texas A&M University
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Tired of pulling out worksheets day after day? Try these activities that build conceptual understandings and reinforce the TEKS. Students need to develop mathematical power not just the ability to circle a, b, c, or d answers. Using activities that encourage students to work in groups and communicate about mathematics is a skill that is missing from worksheet-driven classrooms (Slavin, 1990; Webb, 1989). Visit the www.nctm.org website and click on "electronic activities", where you will find a wealth of activities that students can interact with through the computer. If technology is not easily accessible at your school-site, here are a few activities that will actively involve students in graphing and learning about measures of central tendency while having fun. Let’s concentrate on these activities in data analysis that promote student thinking without using paper and pencil. These activities develop vocabulary and cause students to reflect on mathematical concepts and yet address one or more TAAS test objectives. The mean, median, and mode card games (adapted from TAMU mathematics lab) and hundreds beads can be used from 4th to 7th grade with some adaptations.

On the grade 6 TAAS test one of the objectives listed was: Use median, mode, and range to display data (6.10.B). Even at grade 5 students were required to describe characteristics of data presented in table and graphs including the shape and spread of the data and the middle number (median) (5.13B). What better activities to allow students to conceptually understand these skills than playing the mean, median, and mode games? For all three games students use a regular deck of playing cards with the face cards removed. Aces are counted as ones.

The Mean Game
Deal seven cards to each player. The players determine the mean of their cards. The mean for the seven cards dealt is a player’s score. At the end of the round have students exchange score sheets as seen in Table 1 and verify each other’s answers. In case of dispute, have the pair use a calculator to decide who is correct. The player with the highest (or lowest) total after five games, wins the round.

Table 1. The Mean Game: Score Sheet

Table 2: The Median Game: Score Sheet

The Median Game
Deal seven cards to each player. The players arrange their cards from lowest to highest and find the middle card in the hand. This is the median for the seven cards. Players record this number on the score sheet as seen in Table 2 and then repeat the game five times. At the end of the fifth game, players add their median scores for each game. The player with the highest score wins the median game. As the teacher circulates, he/she asks students questions concerning the range of cards that was dealt to them. The teacher may confirm if they ordered their cards from largest to smallest or smallest to largest and if there was any difference in the median.
The Mode Game
The rules for the mode game are the same as for the median game except for the scoring. For this game, the mode for the seven cards dealt is a player’s score. If a player has more than one mode, the highest mode is the player’s score for that game. If a player does not have at least two of any number there is not mode. Students record their modes on the score sheet as shown in Table 3.

Table 3. The Mode Game: Score Sheet

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Hundred Beads Activity: Fractions, Decimals and Converting a Bar Graph to a Circle Graph
Grade 5 TAKS Objective: The student solves problems by collecting, organizing, displaying, and interpreting sets of data and uses models to relate decimals to fractions (5.12A); and in grade 6: sketch circle graphs to display data (6.10C). In this activity students collect data concerning their favorite hamburger, ice cream, or any other area of interest to the class. They place their name on an index card and put it on the board over their favorite flavor making certin that the cards are touching to make a class bar graph. The information will then be transferred to a circle graph and the data is converted to fractions, decimals, and percents (TAKS Objective 5.2 use models to relate decimals to fractions).

Next steps:
1. Look at the information on the class bar graph of ice cream responses and create a table of the results.
2. Give each small group of two to four students one sentence strip, ruler, set of markers and piece of butcher-block paper.
3. Have groups divide the sentence strip into as many sections as responses to the ice cream survey, leaving at least one inch at the beginning and end to attach the sentence strip ends together. Make the divisions as large as possible. (What problems might you have? How can you resolve those problems?)
4. Using markers (one color for each flavor of the ice cream choices), color as many contiguous segments as there are responses on the bar graph.
5. Paste or tape the ends together forming a circle so that only the colored sections are visible.

6. Place the sentence strip circle in the center of the butcher block paper.
7. Estimate the center of the circle and place a dot. Then trace around the circle, marking each place on the sentence strip changes color on the butcher-block paper circle. Remember to label the sections with the ice cream names before you remove the sentence strip circle.
8. Place your 100 beads evenly around the center of the original circle. (It will take all group members to successfully accomplish this task.) It is okay if your beads are larger or smaller than the drawn circle, however, it is important that the beads are equidistant from the center of the drawn circle.
9. Draw as many lines as flavors from the center dot through the lines on the circle out to the beads. Be careful not to move the beads and each group member will need to hold onto the beads so they do not move.
10. Count the number of beads between the lines and write that number in the section. (How are you going to deal with beads that land on the line? How will you decide which flavor it should be with?)
11. Then convert the number of beads in each section to a fraction (number of beads in one section over the total number of beads), to a decimal, and to a percent.
12. The have the students place the number of students responding to each flavor in the numerator and the total number of responses from the bar graph in the denominator to form a fraction. Use a calculator to convert these fractions to decimals.
13. Check the decimals to make sure they match the answers on the circle graph.

Conclusion
The challenge for teachers is to provide opportunities for students in their classes to develop mathematical power by becoming involved in mathematical activities.
in the classroom (NCTM, 2000). These card games and the hundreds beads graph can be useful tools for reaching the destination of test preparedness by providing activities that will not only help construct student understanding but also encourage students to become expressive mathematicians. Assessment drives instruction and these hands-on activities can equip students with the skills needed to not only excel on the TAKS but also to be able to have mathematical power in an ever-growing, ever-changing complicated mathematical world.

References:


About the Author
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Book Review
Navigating through Geometry in Grades 3-5

Navigating through Geometry in Grades 3-5 is part of the Navigations series published by NCTM. This book is divided into chapters on shapes, location, transformations, and spatial visualizations. An appendix with blackline masters is included. The book also comes with a handy CD-ROM, which includes not only all the blackline masters but also applets to accompany some of the activities.

Each chapter begins with an overview of the theme. Several different lessons, broken down by grade level for each theme, are featured. Each lesson includes information on goals, prior knowledge, materials and equipment, the learning environment, important geometric terms, and the activity, which includes sections on engaging, exploring, assessing, and extending. The final section of each lesson discusses the next steps in instruction.

I really liked this book. All the activities are original, and I learned quite a bit by reading it. I have already used some of the activities in my classroom. I would recommend this publication to others. –Mrs. Kathleen W. Iwanicki, Allgrove School, East Granby, CT.


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Investigating the Mathematics of Monopoly

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Mrs. Sharon Barrs, Assistant Professor, Georgia Southern University, sbarrs@gasou.edu

Introduction

These activities illustrate a number of areas of mathematical content from the setting of the popular board game Monopoly by Parker Brothers (Hasbro). Topics in this article focus mainly on probability and statistics. However, additional activities can easily extend the content to include series and limits for advanced students. Student centered approaches, encouraged by researchers and professional organizations such as the National Council of Teachers of Mathematics (2000) include exploration, formula application, and pattern observation.

The topics are explored by examining activity sheets with a series of questions. Groups of four students should complete activities during class time; however, the content explored is rich enough to be a unit project or an entry in a math fair competition. Solutions can be examined using multiple methods such as: exploration, application of formulas, and observation of patterns in tables and charts created with the use of technology. The lessons can be adapted to a wide range of grade and ability levels.

Teaching Ideas

During class time, activities are to be completed in groups of four. Group members should divide the following duties: (1) rolling the dice, (2) recording the data, (3) performing calculations and (4) gathering information from other groups. Students should be able to calculate probabilities (simple and compound events) and measures of central tendency (mean, median, and mode).

Instead of rolling the dice during the experiments, a random number generator could be used. For example on the TI-83 calculator, randInt(1, 6, 2) simulates rolling a pair of dice. Similarly, randInt(1, 6, 6) simulates three consecutive rolls, if the numbers are grouped into first pair, second pair and third pair representing the rolls of the dice.

In the board game of Monopoly, a roll of doubles gives you an extra turn. If you throw doubles, you move your token as usual (the sum of the two die) and are subject to any privileges or penalties pertaining to the space on which you land. You retain the dice, throw again and move your token as before. Activity 1A and 1B explore probabilities of rolling doubles, first by simulation and then by calculating the theoretical probabilities. The probabilities are calculated by constructing a sample space with a tree diagram and then using the multiplication rule. NOTE: Empirical probabilities are found by performing an experiment several times and recording the relative frequency of success. Theoretical probabilities are calculated by rules based on equally likely outcomes.

Rolling doubles is not always a good thing. If doubles are thrown three times in succession, the player goes to Jail. Activity 2A considers how often rolling “doubles” land you in Jail. Students will simulate this and then calculate the probability of rolling doubles three times in succession. Teachers should stress the rule of large numbers and talk about the need for a large sample size. If a calculator is used to simulate the rolling of a die, a large sample is not difficult to achieve. Students should be encouraged to use multi-stage events and apply a multiplication rule for independent events.

A player can get out of Jail by (1) throwing doubles on any of the next three turns; (2) using the “Get Out of Jail Free” card; (3) purchasing the “Get Out of Jail Free” card from another player; or (4) paying a fine of $50 before rolling the dice on either of the next two turns. If the player has not thrown doubles by the third turn, he must pay the $50 fine. Activity 2B explores the probability that the player will roll at least one double during the next three turns.

Activity 2B also raises the question of whether a player should go ahead and pay the $50 on the first turn to get out of Jail. At the beginning of the game, it is better to get out of Jail quickly so that the player can purchase property. When the properties are developed, it may be advantageous to stay in Jail and thus avoid landing on another player’s property. Note that rent may still be collected while a player is in Jail.

Activity 3 discusses how long a player should expect to wait (in Jail) to throw doubles. Students roll the dice and count the number of rolls before doubles appear. After collecting the data, students compute empirically the average number of rolls of the dice before getting doubles (approximately six rolls).

A word of caution! Intuitively, this answer may seem to make sense. Since six of thirty-six rolls would be expected to be doubles, a student might expect doubles every six rolls. Therefore, by the time he rolls the dice six times, he would expect to roll doubles for the first time. Although it does have the same numeric answer, unfortunately this is not the question asked. The question stated does not ask if a student should expect to get doubles within the first six rolls but does ask after how many rolls should one expect to get doubles for the very first time. This is a problem about wait time. Sometimes doubles will come quickly and other times it may seem as if you will never roll doubles. “On the
average," how many times will you roll the dice before you get doubles for the very first time?

Activity 3 and 4 are the beginning of more advanced content. While the earlier activities required only simple counting and writing ratios, the activity on “wait time” is modeled by a geometric distribution. It begins with a hands-on experiment and then students use the generated data to compute statistical measures of mean, median and mode. By using a computer or lists on a calculator, the spreadsheet (see Appendix I) can be created or it can simply be distributed for student use. We recommend that students develop it or calculate a few lines of values for themselves so that the entries are more meaningful.

The spreadsheet in Appendix I provides an opportunity for students to see several mathematical concepts demonstrated and approximated. For example, the mean is the expected value and can be found by computing the sum of the N*P(N) column. This is shown to be six rolls. The cumulative probability column provides the opportunity to find the median (where the probability is 50% below and 50% above this roll). The median is approximately four rolls. The mode is the first roll (where the probability is highest). This is expected, since the probability function is shown to be decreasing, something students may only realize by doing a few calculations for themselves. In addition, the spreadsheet shows that the sum of the probabilities is equal to one and provides a setting for talking about converging sequences and series. It is a natural setting to begin a discussion about limits. These are the questions and challenges presented in Activity 4. It is the intent of the authors to provide a setting where secondary teachers can make meaningful connections between a numerical approach to a very hands-on activity and the formulas and ideas typically included (often with students memorizing formulas out of context) in Algebra II or Pre-Calculus courses.

For this reason, Activity 4 requires more advanced mathematics than previous worksheets. The table should be generated using a spreadsheet or the list feature of a calculator. By using formulas for geometric series and sequences, students can compute the values theoretically or confirm the values by the patterns they observe in the columns. Technology and the observation of patterns make this problem accessible to many students for whom the necessary advanced mathematics is not yet available (e.g., middle grades students or high school students who have not yet completed Algebra II).

References

Related Articles and Websites
Official Monopoly web site, www.monopoly.com
Pinzler, Andrew. “Go Directly to Jail.” www.pinzler.com/monopoly.html

Solutions to Activity 1A
1-6. Answers will vary.
7. The group’s estimate since it had more rolls of the dice.

Solutions to Activity 1B
1. \[(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\]

\[
\text{Probability} = \frac{6}{36} = \frac{1}{6}.
\]
2. Not shown due to space considerations. Has 36 branches, one for each element in the sample space shown in question 1.

3. 
   a) \( P(1) \times P(1) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \)
   
   b) \( P(2) \times P(2) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \)

   c) \( P(\text{doubles}) = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{6} = \frac{1}{6} \)

4. Assume the first die is thrown, then the probability that the second die matches the first die is \( \frac{1}{6} \).

Solutions to Activity 2A
1-5. Answers will vary.

6. \( P(\text{doubles}) \times P(\text{doubles}) \times P(\text{doubles}) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216} = 0.5\% \)

Solutions to Activity 2B
1. Answers will vary.

2. The probability of rolling doubles, \( P(D) \), is \( \frac{1}{6} \).

   The probability of NOT rolling doubles, \( P(N) \) is \( 1 - \frac{1}{6} = 5/6 \).

   \[ P(\text{NNN}) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216} \approx 58\% \]

3. \( P(\text{at least one double}) = 1 - P(\text{NNN}) = 1 - \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{91}{216} \approx 42\% \)

   Alternately, \( P(\text{at least one double}) = \text{the probability of getting doubles exactly once in the next 3 rolls + doubles exactly twice in the next 3 rolls + doubles for all three rolls.} \)

   \[ \frac{\text{P(NDD)} + \text{P(DND)} + \text{P(DDN)}}{2} = \frac{\text{P(NND)} + \text{P(NDN)} + \text{P(DNN)}}{2} + \frac{1}{216} = \frac{91}{216} \]

4. Notice that the probabilities that are calculated in #2 and #3 are for Complementary Events.

5. Consider, for example, that other players may have several hotels on the board located near the jail. You may choose to delay landing on their hotel by choosing to stay in jail.

Solutions to Activity 3
1-3. Answers will vary.

4. 
   a) \( \frac{1}{6} \)
   
   b) \( \frac{5}{6} \times \frac{1}{6} = \frac{5}{36} \)
   
   c) \( \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216} \)

   \[ \frac{1}{6} + \left( \frac{5}{6} \times \frac{1}{6} \right) + \left( \frac{5}{6} \times \frac{5}{6} \right) = \frac{91}{216} \approx 42\% \]

5. 

6. 6, see data from question #1. Theoretically, see Activity Sheet 4 #9.
Solutions to Activity 4

1. See spreadsheet in Appendix I. The purpose of the spreadsheet is to provide an electronic way for students to do several calculations that would otherwise be very time consuming. For more advanced students who have studied geometric sequences and series, the computations should be performed and the answers confirmed on the table. The table explores the probability that DOUBLES will be rolled for the FIRST TIME on a given roll. The columns represent the following:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>The current roll of dice</td>
<td>P(N)</td>
<td>The probability that doubles are rolled on the Nth roll</td>
</tr>
<tr>
<td>N*P(N)</td>
<td>This column is being constructed because the formula for finding the expected value is known to be the sum of the different values from the sample space times its respective probability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cum P</td>
<td>This column is created so that one can see when the probability of rolling doubles is less than some number of rolls and the probability that it will take more than that number of values to see doubles for the first time. It also confirms that the sum of the probabilities for all of the values in the sample space is ONE.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Template for Excel spreadsheet. The fill down command is very helpful.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>N</td>
<td>P(N)</td>
<td>N*P(N)</td>
<td>Cum P</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>=((5/6)^(A3-1))/6</td>
<td>=A3*B3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>=((5/6)^(A4-1))/6</td>
<td>=A4*B4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>=((5/6)^(A5-1))/6</td>
<td>=A5*B5</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>=((5/6)^(A6-1))/6</td>
<td>=A6*B6</td>
</tr>
<tr>
<td>42</td>
<td>40</td>
<td>=((5/6)^(A42-1))/6</td>
<td>=A42*B42</td>
</tr>
<tr>
<td>43</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>Totals</td>
<td>=SUM(B2:B43)</td>
<td>=SUM(C2:C43)</td>
</tr>
</tbody>
</table>

2. Completing the P(N) column:

\[ P(1) = \frac{1}{6} \]

\[ P(2) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36} \]

\[ P(3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216} \]

\[ P(N) = \frac{1}{6} \times \left( \frac{5}{6} \right)^{N-1} \]

The probability that doubles are rolled on the first roll. See Activity Sheet 1B.

The probability that doubles are rolled for the first time, on the second roll. For this event to occur, doubles WOULD NOT have been achieved on the first attempt, and WOULD have been achieved on the second roll.

The probability that doubles are rolled for the first time, on the third roll.

By examining the first few entries in this column, a pattern emerges and is a geometric sequence with

\[ a = \frac{1}{6} \text{ and } r = \frac{5}{6} \]
3. Decreasing since a geometric sequence with common ratio less than 1.

4. 

\[
\frac{a}{1-r} = \frac{1}{6} = 1 \quad 1 - \frac{5}{6}
\]

5. The sum of all of the probabilities (mutually exclusive events) of the sample space should equal 1.

6. \( N = 1 \), obvious since \( P(N) \) is decreasing.

7. \( N = 4 \), found by examining the last column of the table and finding the first roll when the probability that doubles appears on this roll or any previous roll is at least 0.5.

8. This is the standard sum of the \( N \times P(N) \) column.

\[
\sum_{N=1}^{\infty} N \times P(N) = 6
\]

ROLLING DOUBLES

ACTIVITY 1A

EMPIRICAL PROBABILITY

In the board game of Monopoly, a roll of doubles gives you an extra turn. If you throw doubles, you move your token as
ROLLING DOUBLES ACTIVITY 1A
EMPIRICAL PROBABILITY

In the board game of Monopoly, a roll of doubles gives you an extra turn. If you throw doubles, you move your token as usual (the sum of the two dice) and are subject to any privileges or penalties pertaining to the space on which you land. You retain the dice, throw again and move your token as before.

1. Roll the dice 20 times. Record the number of times doubles occurred.

<table>
<thead>
<tr>
<th>Roll</th>
<th># Doubles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
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<td>10</td>
<td>11</td>
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<td>11</td>
<td>12</td>
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<td>13</td>
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<td>13</td>
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<td>14</td>
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<td>15</td>
<td>16</td>
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<td>17</td>
<td>18</td>
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<tr>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

2. Estimate the probability of rolling doubles on a single throw of the dice by writing the relative frequency from the trials in this experiment.

\[
\frac{\# \text{ Doubles}}{\# \text{ Rolls of Dice}} = \]

3. Gather data from the other groups working on this problem. Record the data in the chart.

<table>
<thead>
<tr>
<th>Group #</th>
<th># Doubles Rolled</th>
<th># Rolls</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Using all of the available data from the class, the estimate for the probability of rolling doubles in a single throw is:

\[
\frac{\text{Total \# of Doubles}}{\text{Total \# Rolls of Dice}} = \]

5. Compare your group’s estimate with the class estimate.
Is your estimate: _______ higher _______ lower _______ equal to the class?

6. If the estimate of the group and the estimate of the class for rolling doubles on a single throw of the dice are not equal, record at least two ideas that your group believes may explain the difference.

7. Also indicate which estimate your group believes is closest to the true probability of getting doubles on a single throw of the dice. Include a reason, if you have one.
ROLLING DOUBLES
THEORETICAL PROBABILITY

In the previous activity, you found the empirical probability of rolling doubles in a single throw of dice. In this activity, you will calculate the theoretical probability of rolling doubles.

1. A method that can be used to calculate the probability of an event is to construct an equally likely sample space. Construct an equally likely sample space for rolling two dice and calculate the probability of rolling doubles.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1,1)</td>
<td>(1,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(2,1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(3,1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>5</td>
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<td></td>
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</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Die 1

Die 2

2. A tree diagram is sometimes used to represent multi-stage events. Construct a tree diagram for rolling two dice and show that it contains the same outcomes as the sample space found in question 1.

3. Another method that can be used to calculate the theoretical probability with rules is to consider that doubles can occur with any number on the dice.
   a. Determine the probability of rolling double one’s. This is the probability of rolling a one on the first die and a one on the second die.
   b. Determine the probability of rolling double two’s.
   c. Decide on an appropriate value for the probability of rolling doubles for any number.

4. Rules are also used to represent theoretical probabilities. Assume that the first die is rolled. What is the probability that the second die matches the first die? Hint: You may want to look at the 6 different possibilities. For example, what if you roll a 1 on the first die?
GO TO JAIL

In the game of Monopoly, a player goes to Jail when (1) his token lands on the space marked “Go to Jail”; (2) he draws a card marked “Go to Jail”; or (3) he throws doubles three times in succession. We are interested in how often rolling “doubles” land you in Jail.

1. Look at the data that was recorded on Activity #1A for rolling doubles. Are there any instances when doubles were rolled three times in succession? Consult the other groups in the class.

2. While one student records data, the remaining three students in the group should all have a pair of dice. For round 1, all three students should roll their dice. Record whether each student rolled doubles. Did all three students get doubles? Continue this twenty times.

3. Did all three students ever get doubles on the same round?

4. Did any group in the class have doubles for all three students on the same round?

5. Use the class information to estimate the probability that doubles can be rolled three times in succession.

6. Use the rules of probability to determine the theoretical probability of rolling doubles three times in succession. Hint: Doubles, then doubles again and then doubles again.

7. Compare your group’s estimate to the class estimate. Which is closer to the theoretical probability calculated in #6?
GET OUT OF JAIL

ACTIVITY 2B

A player can get out of Jail by (1) throwing doubles on any of the next three turns, (2) using the “Get Out of Jail Free” card, (3) purchasing the “Get Out of Jail Free” card from another player, or (4) paying a fine of $50 before rolling the dice on either of the next two turns. A player who throws doubles on any of his three turns would immediately move forward the number of spaces shown on the dice but would forfeit the extra roll of the dice normally awarded for rolling doubles. If the player has not thrown doubles by the third turn, he must pay the $50 fine and immediately move forward the number of spaces shown by his roll of the dice.

1. Approximate the probability that you roll a double anytime during the next three rolls. Use the data from Sheet 2A for the 20 rounds and determine which rounds had at least one double during the round.

2. Compute the theoretical probability that you DO NOT roll doubles during three consecutive throws of the dice.

The events in the sample space for rolling a pair of dice three times and recording Doubles (D) or No Doubles (N) are shown below. Calculate the probability for each event. The event NNN can be recorded as your answer to #2 above.

\[
\begin{align*}
\text{P}(\text{NNN}) &= \\
\text{P}(\text{NND}) &= \\
\text{P}(\text{NDN}) &= \\
\text{P}(\text{DNN}) &= \\
\text{P}(\text{DN}) &= \\
\text{P}(\text{DDN}) &= \\
\text{P}(\text{DND}) &= \\
\text{P}(\text{DDD}) &=
\end{align*}
\]

3. Use the information above to determine the theoretical probability of rolling at least one double during the next three rolls? How does this answer compare with your empirical probability found in question 1?

4. Compare your answers to #2 and #3. Why should these probabilities have a sum of one?

5. Can you think of any advantages or disadvantages to staying in Jail until you roll doubles? Would it always be better to pay the $50 and get out of Jail on your first roll?
HOW LONG DOES IT TAKE TO ROLL DOUBLES?  

When a player is in Jail, one question that is sometimes pondered is how long to wait to get out of Jail. After all, if $50 is going to be paid anyway to get out, why not just pay it on the first roll. Consider the question, “how long should a player expect to wait to roll doubles?”

1. Working in pairs, students should roll a pair of dice until doubles appear. For each experiment, record the number of rolls that were needed to achieve the first roll of doubles (count the roll when the doubles first appear). Repeat the experiment until there are twenty pieces of data.

<table>
<thead>
<tr>
<th>Round</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
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<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td># Rolls to Doubles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Using the 20 pieces of data, compute the mean, median, and mode.

   Mean ___________  Median ___________  Mode ___________

3. The number of times that a player should expect to roll dice before getting doubles is:

4. Using rules that we have learned, determine the theoretical probabilities below.
   a) What is the probability that a player will get doubles on the first roll of the dice?

   b) What is the probability that a player will get his first doubles on the second roll of the dice? Hint: The player would not get doubles on the first roll.

   c) What is the probability that a player will get his first doubles on the third roll of the dice? Hint: The player would not get doubles on the first or second roll.

5. What is the probability that the player will get his first doubles within three consecutive turns to get out of Jail? Sheet 2B may be helpful.

6. Suppose that a modified version of the game permits a player to wait in Jail as long as he wants, but must leave Jail when doubles are rolled. How long should one expect to wait before rolling doubles? If you are not anxious to leave Jail, how long could you expect to stay there?
HOW LONG DOES IT TAKE TO ROLL DOUBLES?  

ACTIVITY 4  

ADVANCED MATHEMATICS

Let us define a probability function, \( P(N) \), to be the probability that the player will roll doubles for the first time on the \( N \)th consecutive throw of the dice.

1. On a separate sheet of paper, create a table that displays several columns: \( N \), the number of the roll; \( P(N) \), the probability of getting doubles for the first time on that roll; and \( N \times P(N) \), the product of the first two columns.

2. What is the probability that a person will get their first doubles on the \( N \)th roll? HINT: The probability function can be represented by an infinite geometric sequence.

3. The probability function, \( P(N) \), is

   increasing       decreasing       non-increasing       non-decreasing

4. Using the table, find the sum of the \( P(N) \) column. What value does this sum approach?

5. Find the sum of \( P(1) + P(2) + P(3) +... \) analytically by computing the sum of an infinite geometric series.

6. What is the significance of the value of the sum?

7. Use the table to find the mode number of rolls for doubles to occur. This is the number \( N \) where the first occurrence of rolling doubles is most likely.

8. Use the table to find the median of the number of rolls for doubles to occur. This is the point where the probability of rolling doubles in less than \( N \) rolls is 50% and the probability of rolling doubles in more than \( N \) rolls is 50%.

9. Use the table to find the mean (expected value) of the number of rolls for doubles to occur. The expected value is \( 1 \times P(1) + 2 \times P(2) + 3 \times P(3) +... = \text{sum of the } N \times P(N) \text{ column.} \) What value does this approach?
### APPENDIX I

<table>
<thead>
<tr>
<th>N</th>
<th>P(N)</th>
<th>N*P(N)</th>
<th>Cum P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1666666667</td>
<td>0.1666666667</td>
<td>0.1666666667</td>
</tr>
<tr>
<td>2</td>
<td>0.1388888889</td>
<td>0.2777777778</td>
<td>0.3055555556</td>
</tr>
<tr>
<td>3</td>
<td>0.1157407407</td>
<td>0.3472222222</td>
<td>0.4212962969</td>
</tr>
<tr>
<td>4</td>
<td>0.0964506061</td>
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</tr>
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<td>0.0803755514</td>
<td>0.4018775728</td>
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<td>0.0323011176</td>
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<td>0.9065361210</td>
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<td>0.0155773131</td>
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<tr>
<td>15</td>
<td>0.0129810944</td>
<td>0.1947164150</td>
<td>0.9350945287</td>
</tr>
<tr>
<td>16</td>
<td>0.0108175797</td>
<td>0.1730812570</td>
<td>0.9459121079</td>
</tr>
<tr>
<td>17</td>
<td>0.0090146499</td>
<td>0.1532490300</td>
<td>0.9549267565</td>
</tr>
<tr>
<td>18</td>
<td>0.0075122207</td>
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<td>0.9624389634</td>
</tr>
<tr>
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<td>0.9686991366</td>
</tr>
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<td>20</td>
<td>0.0052168111</td>
<td>0.1043362133</td>
<td>0.9739159477</td>
</tr>
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<td>0.0043473424</td>
<td>0.0912941870</td>
<td>0.9782632894</td>
</tr>
<tr>
<td>22</td>
<td>0.0036227855</td>
<td>0.0797012743</td>
<td>0.9818860742</td>
</tr>
<tr>
<td>23</td>
<td>0.0030189888</td>
<td>0.0694367160</td>
<td>0.9849050602</td>
</tr>
<tr>
<td>24</td>
<td>0.0025158234</td>
<td>0.0603797534</td>
<td>0.9874208857</td>
</tr>
<tr>
<td>25</td>
<td>0.0020965191</td>
<td>0.0524129800</td>
<td>0.9895174042</td>
</tr>
<tr>
<td>26</td>
<td>0.0017470999</td>
<td>0.0454245832</td>
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</tr>
<tr>
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<td>0.0014559167</td>
<td>0.0393097351</td>
<td>0.9927204199</td>
</tr>
<tr>
<td>28</td>
<td>0.0012132633</td>
<td>0.0339713763</td>
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**Totals** | **0.999319622** | **5.96870262**
Lone Star News

CAMT & NCTM

The Conference for the Advancement of Mathematics Teaching (CAMT) is scheduled for July 17-19, 2003 at the George R. Brown Convention Center in Houston. Please see www.tenet.edu/camt in February for registration and program information.

The annual conference for the National Council of Teachers of Mathematics (NCTM) begins April 9, 2003 at the Henry B. Gonzalez Convention Center in San Antonio. The final day of the conference is Saturday April 12. Please see www.nctm.org for registration and program information.

Affiliate Group News

These are local affiliated groups in Texas. If you are actively involved with them, please send future meeting and conference information to cschneider@mail.utexas.edu so we may publicize your events. Contact information for each group is available on the NCTM website, www.nctm.org.

Alamo District CTM
Austin Area CTM
Big Country CTM
East Texas CTM
Fort Bend CTM
Greater Dallas CTM
Greater El Paso CTM
Houston CTM
Rio Grande Valley CTM
Texas South Plains CTM
Trinity-Brazos River Valley CTM
West Texas Frontier CTM
1960 Area Mathematics Council

Presidental Awards

Four Texas Teachers were among the recipients of the 2001 Presidential Awards for Excellence in Mathematics and Science Teaching. The winners are Patricia Alexander, Barbara Bush Elementary in Grand Prairie, elementary mathematics; D’Ann Douglas, Sallie Curtis Elementary in Beaumont, elementary science; Kristine Smith, Weatherford High, secondary mathematics; and Peggy Schweiger, Klein Oak High in Spring, secondary science. Each 2001 winner receives a $7500 grant for the school and a trip to the ceremony in Washington. The applications may be downloaded from: www.ehr.nsf.gov/pres_awards/

Reviewers

Thank you to our reviewers:
Bill Jasper
Sharon Taylor
Kathy Mittage
Ullrich Reichenbach
Jacqueline Weilmuenster
Paula Gustafson
Jo Ann Wheeler
Beverly Anderson
Diane McGowan
Jim Telese
Sheryl Roehl
Judy Rice
Linda Antinone
Cynthia Schneider

Scholarship Donors

TCTM would like to thank Osborne Allen Brines II for his scholarship donation. You, too, can donate to the TCTM scholarship fund by completing the appropriate section on the membership form.
Book Review

Navigating through Algebra in Grades 6-8

Navigating through Algebra in Grades 6-8 is the first in NCTM’s new Navigation series. This publication is divided into four chapters, each of which concentrates on one component of the Algebra strand in the Principles and Standards for School Mathematics. The Chapters are “Understanding Patterns, Relations and Functions”; “Analyzing Change in Various Contexts”; “Exploring Linear Relationships”; and “Using Algebraic Symbols.” Each chapter begins with a short discussion of the importance of the mathematics idea presented and the level of existing knowledge that students should have before beginning the activities. Activities are presented with blackline masters to model the “big ideas” of algebra. A pre-assessment activity helps in determining what the students’ knowledge is before beginning other activities in the chapter. The goal of all the activities is to develop students’ understanding of “relationships among quantities and the way quantities may change relative to one another.” Students deepen their understanding of algebra by solving these concrete problems. Several of the activities integrate spreadsheet and graphing-calculator technology. The accompanying CD-ROM has two Java applets that allow students to manipulate linear regression graphs by changing parameters.

All the activities are easy to implement. The book provides hands-on examples for middle-level algebra concepts, such as slope and equation writing, to help students analyze the relationships between two quantities. Students will enjoy exploring the bouncing tennis ball activity, as well as writing stories to go with graphs. The activities help students develop abstract thinking by providing realistic problems and establishing a progression from using manipulatives, to creating tables and graphs, to writing equations.—Leslie Knecl, Jefferson Middle School, Champaign, IL 61821.


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Navigating through Geometry in Grades 9-12

Over the past few years, the subject matter of standard high school courses in geometry has been transformed. Additionally, the way that the material is taught has also undergone quite a change. NCTM has published several sets of standards for mathematics teachers, Curriculum and Evaluation Standards for School Mathematics in 1989, Professional Standards for Teaching Mathematics in 1991, Assessment Standards for School Mathematics in 1995, and Principles and Standards for School Mathematics in 2000.

When Curriculum and Evaluation Standards for School Mathematics was published, many teachers (including me) believed that the Standards were nice, but they did not help us teach. Although the book said that transformations, rotations, and so on, should be the underpinning of geometry, we wondered what transformations were and how we could teach concepts about which we knew little. Workshops, of course, followed, as did a lot of manipulatives. Still, we wondered how we could tie this stuff in.

We finally have part of the answer. Developed to provide support in implementing Principles and Standards for School Mathematics in the classroom, this book is not long enough, but it is a blessing. Working in small increments, this book starts by explaining the Standards and then provides the needed definitions and types of exercises that take students and teachers through some of the Standards. The authors tell us early in the book that they had to pick and choose the Standards and materials to concentrate on. I read that statement but did not think much about it until I arrived at the sudden end of the book, on page 74. After that, pages appear that can be reproduced for lessons. These pages help teachers understand the various topics.

A CD-ROM is included in the book, but I had difficulty with it. I easily loaded it into my computer but was able to access only part of chapter 1. Nothing happened when I clicked on the other chapter headings. I found no detailed instructions for using the CD-ROM, just that it included Adobe Acrobat Reader 4 as a plug-in, with no instructions on how to load it if needed. Even with the CD-ROM problems, this book is one that should be on every teacher’s desk.—Ira Lee Riddle, Pennsylvania State University–Abington Campus, Abington, PA 19001, and Montgomery County Community College.


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CAMTERSHIP APPLICATION

Five $200 CAMTerships will be awarded to those teaching five or fewer years who are members of TCTM and have not attended CAMT before. The money is intended to help cover expenses associated with attending CAMT and to encourage new teachers to attend CAMT. Two CAMTerships will be awarded to teachers in each of the following grade levels: K - 4, 5 - 8, and 9 - 12. Winners will be determined by random drawing of names and will be notified by June 1, 2003. Winners will be asked to work for two hours at registration or NCTM material sales and will be TCTM's guest at our breakfast, where the checks will be presented. Good luck!

Deadline: May 1, 2003

Name: __________________________________________
Phone number: ______________________________
Home address: _____________________________________
City, zip: _____________________________
School: __________________________________________
Grade(s) taught: _______________________
School address: __________________________________________
School phone: ________________
Principal's name: _______________________________

Are you a member of TCTM? ___________

Note: If you are not a member of TCTM, you must enclose $13 with this application to apply for membership.

Have you attended CAMT before? _____________

How long have you been teaching? _____________
Describe your primary teaching responsibilities

Send your completed application to: Cynthia Schneider
234 Preston Hollow
New Braunfels, TX 78132
TCTM Leadership Award Application

The TCTM Leadership Award is presented to a TCTM member who is nominated by a TCTM Affiliated Group. This person is to be honored for his/her contributions to the improvement of mathematics education at the local and state level. He/she has designed innovative staff development and has promoted the local TCTM Affiliated mathematics council. **Deadline: May 1, 2003**

**Information about the Affiliated group nominating a candidate:**

Name of Affiliated Group: ____________________________________________________________

President of the Affiliated Group: _______________________________________________________

Home address: _____________________________________________________________________

Home phone: _______________ Business phone: ___________ E-mail: ___________________

Are you a member of TCTM? _________ NCTM? __________

**Information about the person being nominated:**

Name: ____________________________________________________________________________

Home address: _____________________________________________________________________

Home phone: _______________ Business phone: ___________ E-mail: __________________

Is the nominee a member of TCTM? _________ NCTM _________ Retired ________

Applications should include 3 pages:

- Completed application form
- One-page, one-sided, typed biographical sheet including:
  - Name of nominee
  - Professional activities
  - State/local offices or committees
  - Activities encouraging involvement/improvement of math education
  - Staff Development
  - Honors/awards

- One-page, one-sided essay indicating why the nominee should be honored for his/her contribution to the improvement of mathematics education at the state/national level.

**Send the completed application, biographical sketch, and essay to**

Cynthia Schneider
234 Preston Hollow
New Braunfels, TX 78132
E. Glenadine Gibb Achievement Award Application

The E. Glenadine Gibb Achievement Award is presented to someone nominated by a TCTM member to be honored for his/her contribution to the improvement of mathematics education at the state and/or national level. **Deadline: May 1, 2003**

**Information about the TCTM member nominating a candidate:**

Name: ______________________________________________________________________________

Home address: ________________________________________________________________________

Home phone: _______________ Business phone: ______________ E-mail: ________________

Are you a member of TCTM? ____________ NCTM? ______________

**Information about the nominee:**

Name: ______________________________________________________________________________

Home address: ________________________________________________________________________

Home phone: _______________ Business phone: ______________ E-mail: ________________

Is the nominee a member of TCTM? _________ NCTM? __________ Retired _________

Applications should include 3 pages:

- Completed application form
- One-page, one-sided, typed biographical sheet including:
  - Name of nominee
  - Professional activities
  - National offices or committees
  - State TCTM offices held
  - Local TCTM-Affiliated Group offices held
  - Staff Development
  - Honors/awards
- One-page, one-sided essay indicating why the nominee should be honored for his/her contribution to the improvement of mathematics education at the state/national level

Send the completed application, biographical sketch, and essay to:

Cynthia Schneider
234 Preston Hollow
New Braunfels, TX 78132
TEXAS COUNCIL OF TEACHERS OF MATHEMATICS
MATHEMATICS SPECIALIST SCHOLARSHIP

Amount: $1000

Application Deadline: May 1, 2003

Eligibility: Any student who will graduate in 2003 from a Texas high school - public or private - and who plans to enroll in college in Fall 2003 to pursue a career in mathematics teaching either as a mathematics specialist in elementary school or as a secondary school teacher with certification in mathematics.

Name: ____________________________________________________________

                        Last   First   Middle
Address: __________________________________________________________

                        Number and street   Apt. number

                        City   Zip code

Phone number: ( )__________________  Birth date: ____________________________

Social security number: ________________________________

High school(s) attended: ______________________________________

What college or university do you plan to attend? If you are awarded this scholarship, TCTM’s treasurer will send a check directly to the business office of the college. We need the college’s complete address.

Enclose the completed application with each of the following in the same envelope and mail to Pam Alexander at the address listed below. YOU MUST INCLUDE 3 COPIES OF ALL REQUIRED MATERIALS.

1. On a separate sheet, list high school activities including any leadership positions.
2. Official high school transcript
3. Letter of recommendation from a TCTM member
4. An essay describing your early experiences learning mathematics and any experiences explaining mathematics to your classmates or friends. This essay must be no more than two pages, double-spaced.
5. An essay telling why you want to be a mathematics specialist in elementary school or a mathematics teacher in middle or high school. This essay must be no more than one page, double-spaced.

Return all materials in one envelope to:

Cynthia Schneider
234 Preston Hollow
New Braunfels, TX 78132
TCTM Region   ESC Regions
Southwest     15, 18, 19
Southeast     4, 5, 6
Northwest     9, 14, 16, 17
Northeast     8, 7, 10, 11
South         1, 2, 3
Central       12, 13, 20
When does YOUR membership expire?

Note the expiration date on your mailing label. Use the membership form inside to renew before that date.

Texas Council of Teachers of Mathematics
Member 2002-2003

NAME ___________________________

Texas Mathematics Teacher
234 Preston Hollow
New Braunfels, TX 78132

Change Service Requested